

3.2 Compound & Double Angle Formulae

Question Paper

Course	CIEA Level Maths
Section	3. Trigonometry
Topic	3.2 Compound & Double Angle Formulae
Difficulty	Hard

Time allowed: 80
Score: /60
Percentage: /100

Question 1

If $A = B$, then

$$\sin(A - B) = \sin(A - A) = \sin(0) = 0 = \sin A - \sin A = \sin A - \sin B$$

By using a suitable counter-example with $A \neq B$, prove that $\sin(A - B) = \sin A - \sin B$ is **not** true in general.

[2 marks]

Question 2a

(a) Express $\cos(285^\circ)$ in terms of cosines and sines of 315° and 30° .

[2 marks]

Question 2b

(b) Hence show that $\cos(285^\circ) = \frac{\sqrt{6}-\sqrt{2}}{4}$.

[3 marks]

Question 3

Show that

$$\sin 2A \equiv 2 \sin A \cos A$$

(You may use the identity $\sin(A + B) \equiv \sin A \cos B + \cos A \sin B$.)

[2 marks]

Question 4

Show that $2 \cos \theta - 5 \sin \theta$ can be written in the form $R \cos(\theta + \alpha)$, where R and α are constants with $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give R in the form \sqrt{k} where k is an integer, and give α correct to three significant figures.

[5 marks]

Question 5a

(a) Solve the equation

$$\sin 2\theta = \sin \theta \quad -\pi \leq \theta \leq \pi$$

[6 marks]

Question 5b

(b) Solve the equation

$$\cos 2x + \sin^2 x = 0 \quad 0 \leq x \leq 2\pi$$

[4 marks]

Question 6

Show that

$$\frac{\sin(A + B) + \sin(A - B)}{\cos(A + B) + \cos(A - B)} \equiv \tan A \quad \left(A, B \neq \left(k + \frac{1}{2} \right) \pi \right)$$

[4 marks]

Question 7a

(a) Show that $2 \sin \theta + 4 \cos \theta$ can be written as $2\sqrt{5} \cos(\theta - \alpha)$, where $\alpha = 0.464$ to three significant figures.

[4 marks]

Question 7b

(b) Hence solve the equation

$$2 \sin \theta + 4 \cos \theta = 3 \quad -\pi \leq \theta \leq \pi$$

giving your answers correct to 3 significant figures.

[3 marks]

Question 8a

(a) By letting $B = 2A$, use the identity for $\tan(A + B)$ to derive an expression for $\tan 3A$ in terms of $\tan A$.

[5 marks]

Question 8b

(b) Hence, or otherwise, solve the equation

$$\frac{6 \tan x - 2 \tan^3 x}{1 - 3 \tan^2 x} = 2 \quad 0 \leq x \leq \pi$$

[3 marks]

Question 9

Sketch the graph of $y = 2(\sin x - \cos x)$ for $0^\circ \leq x \leq 360^\circ$.

Be sure to label any points where the graph intercepts the coordinate axes, and state the coordinates of any maximum and minimum points.

[7 marks]

Question 10

Show that

$$2 - 2 \cot 2A \tan A \equiv \sec^2 A \quad A \neq k\pi$$

[3 marks]

Question 11a

The alternating voltage, V , in an electrical circuit t seconds after it is switched on is modelled by the function

$$V = 55\sqrt{2} \left(\sin \frac{\pi t}{60} + \cos \frac{\pi t}{60} \right).$$

(a) Express

$$55\sqrt{2} \left(\sin \frac{\pi t}{60} + \cos \frac{\pi t}{60} \right)$$

in the form

$$R \sin \left(\frac{\pi t}{60} + \alpha \right)$$

where R and α are constants to be found. $R > 0$ and α acute.

[3 marks]

Question 11b

(b) Find the voltage when the circuit is switched on.

[2 marks]

Question 11c

- (c) (i) Write down the maximum voltage and the time at which this first occurs.
(ii) Find the time it takes the voltage to complete one period (cycle).

[2 marks]